CS 4820

Homework 9

Problem 2

**Part A**

In the graph, the maximum value matching uses the two edges (A,B) and (C,D) which has a total value 1 + 1 = 2. The greedy algorithm considers edges in the order (A,D), (A,B), (C,D), takes edge (A,D) and then can take not other edge, resulting in a total value of only 1. Hence, the greedy algorithm finds a matching that has a factor of 2 smaller value than the maximum possible.

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**Part B**

Consider the greedy matching M and the maximum matching M\*. Let Y be the side that is given, and nodes on the X side arrive one-at-a-time. When the greedy algorithm selects an edge e, there are three possible cases associated with e:

1. e ∈ M and e ∈ M\*
2. There is exactly one edge e’ ∈ M\* such that e’ shares an end node with e. As the greedy algorithm chooses edges in decreasing order of value, e has the largest value
   1. Hence, the value of edge e, v(e) > v(e’) > ½ v(e’)
3. There are exactly two edges e’ ∈ M\* and e’’ ∈ M\* that share differing end nodes with e. Again, as the greedy algorithm chooses edges in decreasing order of value this gives,
   1. v(e) > v(e’) and v(e) > v(e’’), which gives
   2. v(e) > ½ (v(e’) + v(e’’))

Calculated the total value of M,

by (2) and (3)

Thus, the greedy algorithm is guaranteed to return a solution that has value at least ½ of the maximum possible value.